The Effect of Contrasting Examples on Transfer in Algebra Problem Solving

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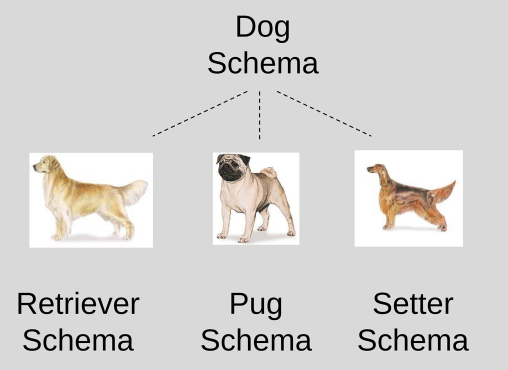
Purpose

The effectiveness of education depends on the transfer of learning. The instructional method of presenting contrasting examples has been hypothesized to enhance transfer, but previous experimenters have not controlled the variable of student study time. If time is controlled, will contrasting examples improve students' ability to transfer an algebra problem solving method to a novel situation?

Background: Experts and Novices

Simplistic Example



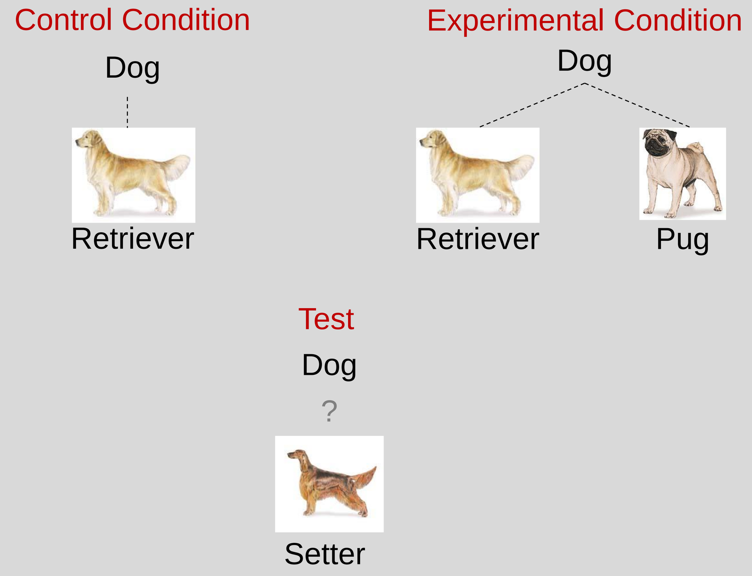
An expert in a particular domain has an enhanced knowledge structure for that domain. Where a novice may perceive three different breeds of dog, the expert sees an integrated pattern. The expert has a hierarchical knowledge structure. In the illustration, the expert *connects the dots*, the novice does not.

A person's ability to transfer knowledge is one indicator of the presence or absence of such a hierarchical knowledge structure. If a learner is applying knowledge of retrievers, pugs, and setters to a new type of dog, like a terrier, that would be evidence that the learner is developing the knowledge structure of an expert. Whether or not the knowledge is applied correctly, the *attempt* to apply the knowledge indicates the learner is connecting the dots.

Do contrasting examples improve transfer?

A reasonable strategy for stimulating a learner to build a hierarchical knowledge structure (and subsequently improve transfer) would be to include contrasting examples in the introductory material.

Experiments have been conducted along these lines. Bransford and Schwartz (1999) found that students studying contrasting examples of psychology experiments were better able to transfer their understanding to interpret new cases. A technical problem with the study was that the students in the experimental condition had more cases to review while studying and presumable spent more time studying the material. Time can not be ignored as a potentially significant variable. Can these results be replicated when students' study time is controlled?



The Substitution Method

Many algebra problems involve a problem within a problem. One technique for solving this type of nested problem is to use a technique called substitution. Substitution is commonly introduced as an advanced technique of integration in the second semester of calculus, but the technique can be applied to some precalculus algebra problems.

When a problem of the form $f(g(x))=0$ is posed, the solution can be split into two parts by substituting for $g(x)$ initially ("Let $t=g(x)$"), and then solving $f(t)=0$. This will yield one or more solutions for t (or perhaps no solutions). Finally, the process is reversed, setting $g(x)=t$ and solving for the original variable of interest, x.

Substitution Example

$$(\log(x-7))^2+\log(x-7)=6$$
$$\text{Let } t=\log(x-7)$$
$$t^2+t=6$$
$$t^2+t-6=0$$
$$(t+3)(t-2)=0$$
$$t+3=0 \quad \text{or} \quad t-2=0$$
$$t=-3 \qquad t=2$$
$$\log(x-7)=-3 \qquad \log(x-7)=2$$
$$10^{-3}=x-7 \qquad 10^{2}=x-7$$
$$0.001+7=x \qquad 100+7=x$$
$$7.001=x \qquad 107=x$$

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The Experiment

Thirty-one beginning calculus students were given a college algebra pretest to control for pre-existing algebra fluency. Students were randomly assigned to study conditions and used workbooks to study the "substitution method" for solving complex algebra and trigonometry problems. Workbooks for the control condition used only examples in which the substitution was performed for a polynomial, e.g. "let $t = x^3$". Workbooks for the contrasting example condition included polynomial substitutions as well as substitutions for trigonometric functions, e.g. "let $t = \sin x$". The posttest was used to determine whether transfer could be detected to problems of the same general type, but calling for different substitutions, e.g. "let $t = \log x$".

Control Condition

Substitution Examples

Let $t = x^3$

Experimental Condition

Substitution Examples

Let $t = x^3$ | Let $t = \sin x$

Test

?

Let $t = \log x$

Results

This experiment confirmed well-established findings that algebraic fluency (measured on the pretest) is the best predictor of transfer in algebra problem solving. Taking this algebraic fluency into account statistically, this experiment found that the correlation between the contrasting example condition and the transfer score was nearly zero ($F_{1,27} = 0.006$, $p \approx 0.94$). This result is not merely "not significant", it is nearly random.

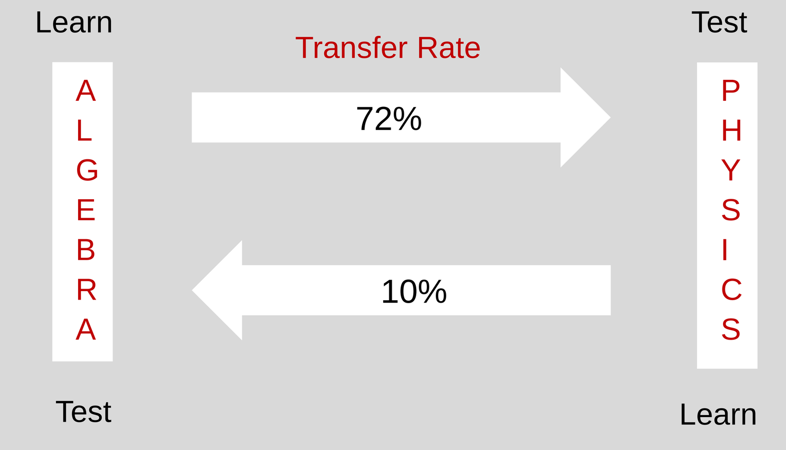
Regression Model

Transfer = 3.6 + 0.152*Pretest + 0.031*ContrExample

Confirmation

These results are unexpected. They contradict intuition as well as the findings of Bransford and Schwartz (1999). A similar non-result can be found in the literature. Bassok and Holyoak (1989) were investigating transfer between mathematics and physics problems that had identical underlying structure (*isomorphic* problems). In their initial experiment, one group was taught using mathematics examples and tested on physics problems, while the other group was taught with physics examples and tested with mathematics problems. They found that the transfer rate from mathematics to physics was much higher than the rate from physics to mathematics. Evaluating their initial experiment, they determined that the mathematics examples they used were examples of contrasting types and believed the contrasting examples might account for the discrepancy. They designed a second experiment with the contrast removed from the mathematics examples, and the results were very similar to their initial experiment. (They continued to alter the experiment and concluded the reason for the discrepancy was discrete phenomena versus continuous phenomena. Learners are more likely assume a formula from a continuous problem can be applied to a discrete problem than the reverse.) They did not directly compare their initial experiment to their second experiment, but the fact that they found the same results with contrasting examples as they found without contrast is support for the unexpected results of the substitution experiment described above.

Bassok and Holyoak (1989)



Conclusions

These results indicate that in the early stages of learning a new problem solving schema, learners' likelihood of transferring that schema to an isomorphic problem is not affected by contrasting examples. In both this experiment and earlier work by Bassok and Holyoak, student study time was limited. It would not be correct to assume that this non-effect for contrasting examples continues though later stages of learning and practicing a problem solving schema.

A large body of literature has addressed distinctions between novices and experts in their particular domain. One characteristic observed in experts is their heightened awareness of details and nuances in problems posed to them. The present experiment seems to mirror those findings in that when a novice is in the earliest phases of learning a new concept, the novice seems to be oblivious to contrasting details. Referring back to the example of the dogs, the learner notices only the tail, four legs, and bark of the dog without noticing the features that make a Pug look very different from a Golden Retriever. This may be a mechanism to mitigate cognitive load during early phases of learning: the novice simply "tunes out" (does not attend to) extraneous information.

Future Research

Study time needs to be investigated as an independent variable. If in the early phases of learning a schema, contrasting examples does not affect transfer, will contrasting examples improve transfer in later stages of learning?